Water Vapor Pressure Formulations

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Saturation vapor pressure formulations

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A large number of saturation vapor pressure equations exists to calculate the pressure of water vapor over a surface of liquid water or ice. This is a brief overview of the most important equations used. Several useful reviews of the existing vapor pressure curves are listed in the references.

1) Vapor pressure over liquid water below 0°C

- Goff Gratch equation [1]
  (Smithsonian Tables, 1984, after Goff and Gratch, 1946):

$$\begin{aligned}\mathrm{Log}_{10}\, e_w = & -7.90298\,(373.16/T-1) \\ & + 5.02808\,\mathrm{Log}_{10}(373.16/T) \\ & - 1.3816\ 10^{-7}\,(10^{11.344\,(1-T/373.16)} - 1) \\ & + 8.1328\ 10^{-3}\,(10^{-3.49149\,(373.16/T-1)} - 1) \\ & + \mathrm{Log}_{10}(1013.246)\end{aligned}$$

  with T in [K] and e_w in [hPa]

- Guide to Meteorological Instruments and Methods of Observation (CIMO Guide) (WMO, 2008) [2]

$$e_w = 6.112\, e^{(17.62\, t/(243.12 + t))}$$

  with t in [°C] and e_w in [hPa]

- WMO [3]
  (Goff, 1957; WMO, 2012):

$$\begin{aligned}\mathrm{Log}_{10}\, e_w = & \ 10.79574\,(1-273.16/T) \\ & - 5.02800\,\mathrm{Log}_{10}(T/273.16) \\ & + 1.50475\ 10^{-4}\,(1 - 10^{(-8.2969*(T/273.16-1))}) \\ & + 0.42873\ 10^{-3}\,(10^{(+4.76955*(1-273.16/T))} - 1) \\ & + 0.78614\end{aligned}$$

  with T in [K] and e_w in [hPa]

- Hyland and Wexler [4]

  (Hyland and Wexler, 1983):

  $$\begin{aligned}\text{Log } e_w = & -0.58002206\ 10^4 / T \\ & + 0.13914993\ 10^1 \\ & - 0.48640239\ 10^{-1}\ T \\ & + 0.41764768\ 10^{-4}\ T^2 \\ & - 0.14452093\ 10^{-7}\ T^3 \\ & + 0.65459673\ 10^1\ \text{Log}(T)\end{aligned}$$

  with T in [K] and e_w in [Pa]

- Hardy [5]

  (Hardy, 1998):

  $$\begin{aligned}\text{Log } e_w = & -2.8365744\ 10^3 / T^2 \\ & - 6.028076559\ 10^3 / T \\ & + 1.954263612\ 10^1 \\ & - 2.737830188\ 10^{-2}\ T \\ & + 1.6261698\ 10^{-5}\ T^2 \\ & + 7.0229056\ 10^{-10}\ T^3 \\ & - 1.8680009\ 10^{-13}\ T^4 \\ & + 2.7150305\ \text{Log}(T)\end{aligned}$$

  with T in [K] and e_w in [Pa]

- Buck [6]

  (Buck Research Manual (1996); updated equation from Buck, A. L., New equations for computing vapor pressure and enhancement factor, J. Appl. Meteorol., 20, 1527-1532, 1981)

  $e_w = 6.1121\ e^{(18.678\ -\ t\ /\ 234.5)\ t\ /\ (257.14\ +\ t)}$ [1996]

  $e_w = 6.1121\ e^{17.502\ t\ /\ (240.97\ +\ t)}$ [1981] [7]

  with t in [°C] and e_w in [hPa]

- Sonntag [8]

  (Sonntag, 1994)

  $$\begin{aligned}\text{Log } e_w = & -6096.9385 / T \\ & + 16.635794 \\ & - 2.711193\ 10^{-2} * T \\ & + 1.673952\ 10^{-5} * T^2 \\ & + 2.433502 * \text{Log}(T)\end{aligned}$$

  with T in [K] and e_w in [hPa]

- Magnus Tetens [9]

  (Murray, 1967)

  $e_w = 6.1078\ e^{17.269388\ *\ (T-273.16)\ /\ (T\ -\ 35.86)}$

  with T in [K] and e_w in [hPa]

- Bolton [10]
  (Bolton, 1980)

  $e_w = 6.112\, e^{17.67 * t / (t+243.5)}$

  with t in [°C] and e_w in [hPa]

- Murphy and Koop [11]
  (Murphy and Koop, 2005)

  $$\begin{aligned}\text{Log } e_w = {} & 54.842763 \\ & - 6763.22 / T \\ & - 4.21 \text{ Log}(T) \\ & + 0.000367\, T \\ & + \text{Tanh}\{0.0415\, (T - 218.8)\} \\ & \cdot\ (53.878 - 1331.22 / T - 9.44523 \text{ Log}(T) + 0.014025\, T)\end{aligned}$$

  with T in [K] and e_w in [Pa]

- International Association for the Properties of Water and Steam (IAPWS) Formulation 1995 [12]
  (Wagner and Pruß, 2002)

  $$\begin{aligned}\text{Log } (e_w/22.064e6) = 647.096/T * ((& -7.85951783\, \nu \\ & + 1.84408259\, \nu^{1.5} \\ & - 11.7866497\, \nu^{3} \\ & + 22.6807411\, \nu^{3.5} \\ & - 15.9618719\, \nu^{4} \\ & + 1.80122502\, \nu^{7.5}))\end{aligned}$$

  with T in [K] and e_w in [Pa] and $\nu = 1 - T/647.096$

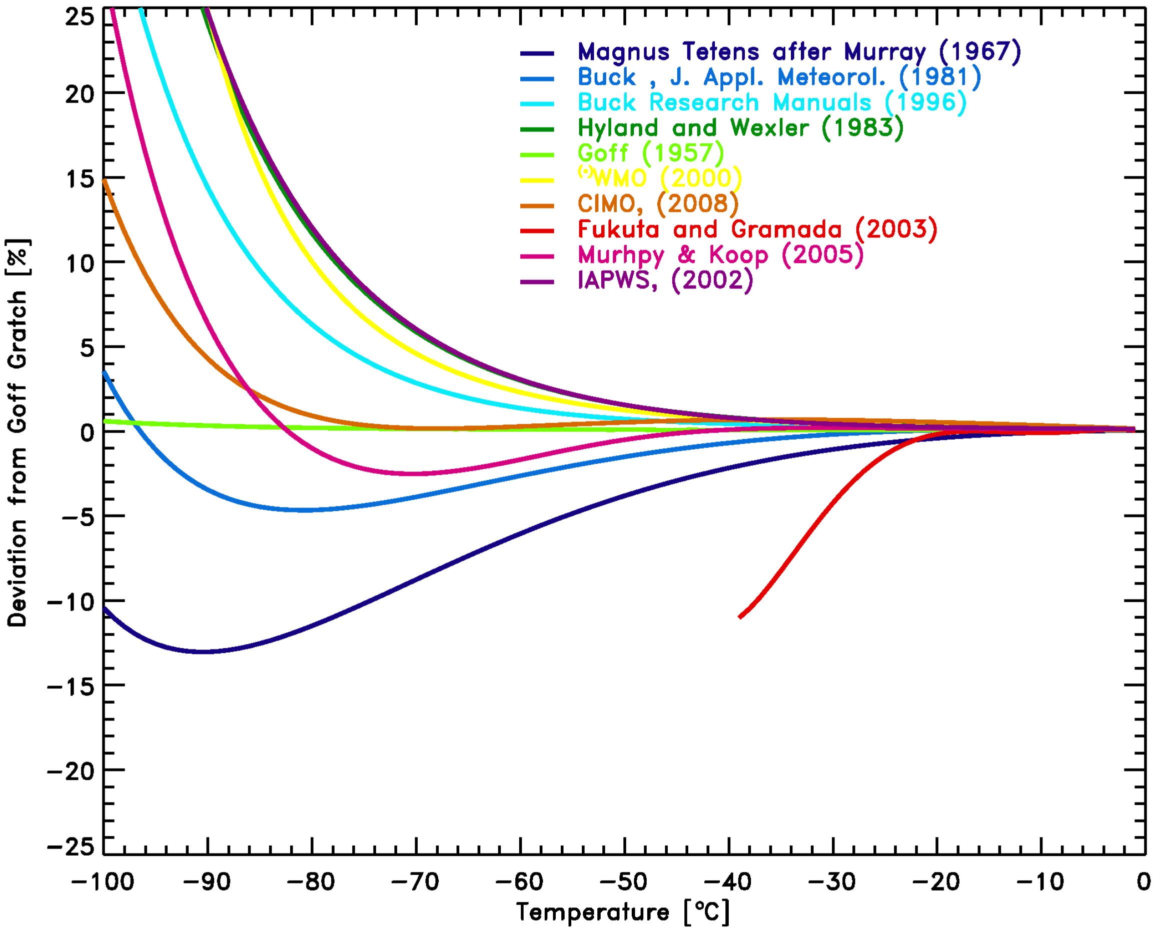

Figure 1: Comparison of equations [2]-[11] with the Goff Gratch equation [1] for the saturation pressure of water vapor over liquid water. The measurements by Fukuta et al. [2003] are shown as well.

Figure 2: Comparison of several equations with the equation by Sonntag [7] for the saturation pressure of water vapor over liquid water.
The equations by Hyland and Wexler [4], the nearly identical equation by Wexler (1976, see reference below) and the equation by Sonntag [7] are the most commonly used equations among radiosonde manufacturers and should be used in upper air applications to avoid inconsistencies.

- Goff Gratch equation
(Smithsonian Tables, 1984):

$$\begin{aligned}\mathrm{Log}_{10}\, e_i = & -9.09718\,(273.16/T - 1)\\ & - 3.56654\,\mathrm{Log}_{10}(273.16/\,T)\\ & + 0.876793\,(1 - T/\,273.16)\\ & + \mathrm{Log}_{10}(6.1071)\end{aligned}$$

with T in [K] and e_i in [hPa]

[13] • Hyland and Wexler

(Hyland and Wexler, 1983.):

$$\text{Log } e_i = -0.56745359\ 10^4 / T + 0.63925247\ 10^1 - 0.96778430\ 10^{-2}\ T + 0.62215701\ 10^{-6}\ T^2 + 0.20747825\ 10^{-8}\ T^3 - 0.94840240\ 10^{-12}\ T^4 + 0.41635019\ 10^1\ \text{Log}(T)$$

with T in [K] and e_i in [Pa]

[14] • Guide to Meteorological Instruments and Methods of Observation (CIMO Guide)

(WMO, 2008)

$$e_i = 6.112\ e^{(22.46\ t/(272.62 + t))}$$

with t in [°C] and e_i in [hPa]

[15] • Magnus Teten

(Murray, 1967)

$$e_i = 6.1078\ e^{21.8745584 * (T-273.16) / (T - 7.66)}$$

with T in [K] and e_w in [hPa]

[16] • Buck

(Buck Research Manual (1996

$e_i = 6.1115\ e^{(23.036 - t / 333.7)\ t / (279.82 + t)}$ [1996] [17]

$e_i = 6.1115\ e^{22.452\ t / (272.55+t)}$ [1981]

with t in [°C] and e_i in [hPa] [18] • Marti Mauersberger

(Marti and Mauersberger, 1993)

> $\text{Log}_{10}\ e_i = -2663.5 / T + 12.537$
> with T in [K] and e_i in [Pa]

[19] • Murphy and Koop

(Murphy and Koop, 2005)

> $$\text{Log } e_i = 9.550426 - 5723.265/T + 3.53068\ \text{Log}(T) - 0.00728332\ T$$
> with T in [K] and e_i in [Pa]

[20] • Sonntag

(Sonntag, 1994)

$$\log e_i = -6024.5282 / T + 24.721994 + 1.0613868 \cdot 10^{-2} * T - 1.3198825 \cdot 10^{-5} * T^2 - 0.49382577 * \log(T)$$

with T in [K] and e_i in [hPa]

[21]

The Goff Gratch equation [13] for the vapor pressure over ice covers a region of -100°C to 0°C. It is generally considered the reference equation; however, other equations have also been widely used. The equations discussed here are mostly of interest for frost-point measurements using chilled mirror hygrometers, since these instruments directly measure the temperature at which a frost layer and the overlying vapor are in equilibrium. In meteorological practice, relative humidity is given over liquid water (see section 1) and care needs to be taken to consider this difference.

Buck Research, which manufactures frost-point hygrometers, uses the Buck formulations in their instruments. These formulations include an enhancement factor, which corrects for the differences between pure vapor and moist air. This enhancement factor is a weak function of temperature and pressure and corrects about 0.5% at sea level. For the current discussion it has been omitted.

The Marti Mauersberger equation is the only equation based on direct measurements of the vapor pressure down to temperatures of 170 K.
The comparison of equations 14-20 with the Goff Gratch equation (Figure 3) shows, that with the exception of the Magnus Teten formula, the deviations in the typical meteorological range of -100°C to 0°C are less than 2.5%, and smaller than typical instrumental errors of frost-point hygrometers of 5-10%.

Not shown is the WMO recommended equation for vapor pressure over ice, since it is nearly identical with the Goff-Gratch equation [13].The equation by Sonntag (1994) agrees with Goff Gratch to within 0.19% over the typical meteorological range of -100°C to 0°C and is not shown.

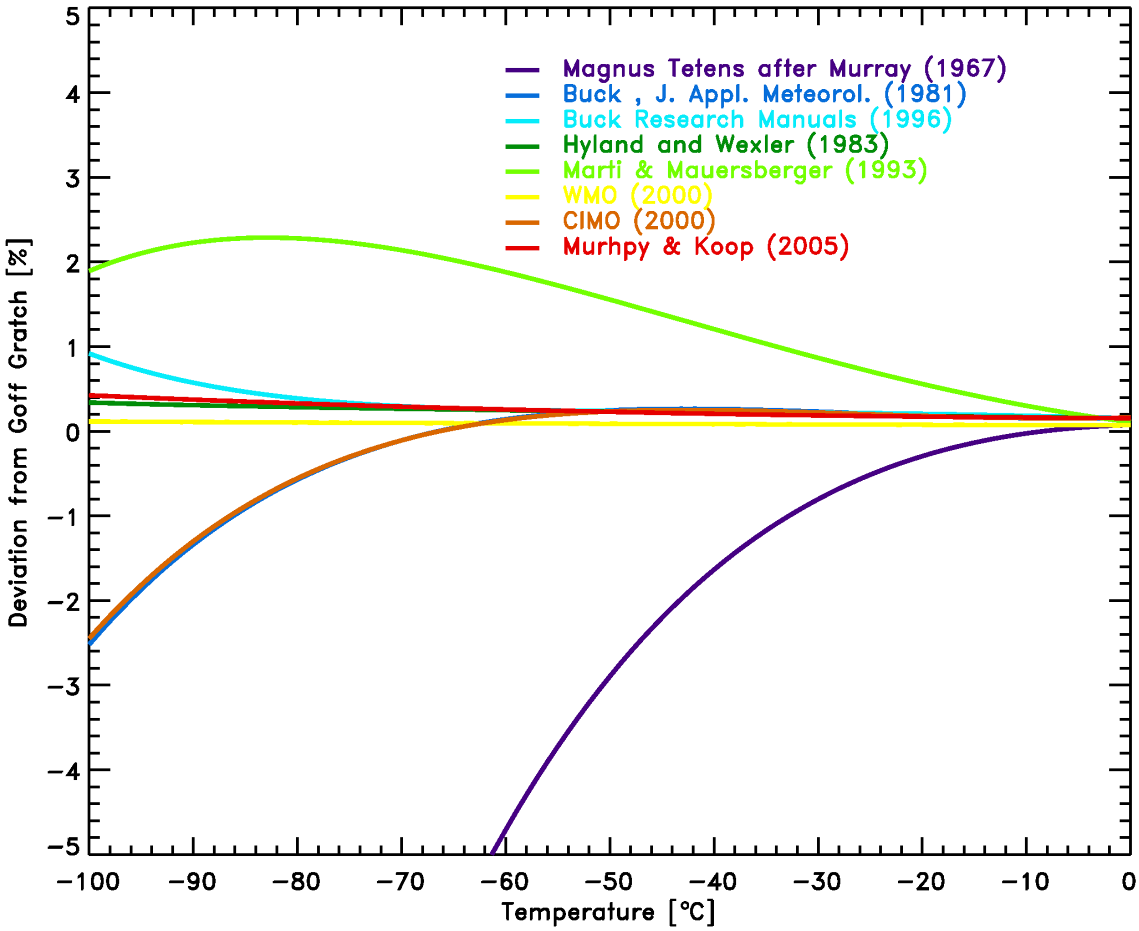

Figure 3: Comparison of equations [14]-[19] with the Goff Gratch equation [13] for the saturation pressure of water vapor over ice.

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