

Water Vapor - Formulas

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There are many equations to compute the saturation vapor pressure of water depending on just the temperature. **mcSaturationVaporPressure.pas** (written in Pascal and included in the [zip file](#)) implements 30 of these.

- 25 are from [Holger Vömel](#) (CIRES, University of Colorado, Boulder).
- The Antoine equation
- The August equation
- Two are used in the GISS Model-E Global Warming simulator
- One is from a text book

Placing these algorithms in a separate module simplifies reusing them in various programs and guarantees that the computations will be the same.

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Calibration Points over Water

Version 1.2 of my programs provide 30 saturation water vapor formulas.

- 11 over ice
- 19 over water

Some produce results in mbar (hPa) and others in mmHg (millimeters of mercury). Some raise 10 to an exponent, and other use base-e. However, these are minor differences and the conversions are fairly trivial.

Many of the formulas have the same (or can be reduced to the same) basic structure, but with different coefficients.

One thing I found interesting is that the calibration points vary. Sometimes a temperature is explicitly used, other times it is the pressure. Occasionally, it is both. The following table captures some of the variations in the "over water" algorithms.

Formula structure	Algorithm	Year	Calibration Temperature	Calibration Pressure	Temp	Comment
$\log(P) = A + B/T$	August_equation	1825	373.1636	1013.25		For GISS_AR4, only the pressure was specified in the formula and the temperature was then computed from that.
	GISS_AR4	??	273.16	6.108		
$\log(P) = A + B/(C+T)$	Antoine_equation	1888	373.1469	1013.25		For the other 2, there are no obvious calibration points - I simply selected 1 atm and computed the temperature.

log(P) = A + B*T/(C+T)	Bolton	1980	273.15	6.112		Both the temperature and pressure are explicitly set Of these, only MagnusTetens uses the triple point, the rest use 0.00°C
	CIMO	2008	273.15	6.112		
	Buck_original	1981	273.15	6.1121		
	MagnusTetens	1967	273.16	6.1078		
log(P) = A + (B*T + C*T^2)/(D+T)	Buck_manual	2001	273.15	6.1121		
Log(P) = 4th order power series	Seinfeld_Pandis	2006	373.15	1013.25		
Complex	MurphyKoop	2005	273.16	6.11657		No obvious calibration points The pressure is simply calculated from the triple point
	Preining	2002	273.16	6.107		
	HylandWexler	1983	273.16	6.11657		
	Sonntag	1994	273.16	6.11657	ITS-90	
	Wexler	1977	273.16	6.0864	IPTS-68	
	IAPWS	1995	647.096	22.064E4	ITS-90	Temperature and Vapor pressure at the critical point
	Fukuta	2003	373.16	1013.246		These values are used in the equation, but the formula does not give a valid result at that temperature - the notes say 373.16 K = steam point
	WMO2000	2000	273.16	6.11139		Pressure computed from temperature These formulas are identical except for the sign of one exponent. WMO2000 is the correct version.
	WMO_Goff	1957				
	GoffGratch	1946	373.16	1013.246		The notes say 373.16 K = steam point temperature

From [Review of the vapour pressures of ice and supercooled water for atmospheric applications](#) by D. M. Murphy and T. Koop, 29 Dec 2006

The vapour pressure of both ice and liquid water at the triple point is

$P_t = 611.657 \pm 0.01 \text{ Pa}$ at temperature $T_t = 273.16 \text{ K}$

(Guildner et al. 1976, Vapor pressure of water at its triple point. J. Res. Natl. Bur. Stand., 80A, 505-521)

Though the relation is not exact, the following is close.

```
273.16 K / 6.11657 mbar -> 273.15 K / 6.112  
mbar
```

Because the standard temperature scale was redefined in 1990, the **current** ([ITS-90](#)) steam point temperature is [373.124 K \(99.974°C\)](#) at 1 atm (1013.25 mbar), **not** 373.15 K (100°C - the old IPTS-68 standard), and **not** 373.16 K (used in 2 of the equations).

Most of the dates come from [Holger Vömel](#). However, those appear to reflect when the coefficients were published, not when the equation was first used. For instance, the MagnusTetens equation was published in 1967, Tetens first published in 1930, and the original Magnus equation dates to 1844. Also, several references provide dates one year earlier than Vömel, not sure why.

[Uncertainty in the generation of humidity](#) (page 22 and Table 12) explains that Wexler (1976 over water, 1977 over ice) used IPTS-68 and that Sonntag (1970) updated Wexler's equations to ITS-90.

Besides the 4 algorithms based on the Magnus equation listed above, "[Improved Magnus` form approximation of saturation vapor pressure](#)" lists additional coefficients and then computes several new coefficients.

Antoine and August equations

The [Antoine and August equations](#) are fairly useless (rough approximations - low accuracy), but of historical importance. I included these because

- There was a [recent Nature article](#) (22 April 2014) that used the Antoine equation
- NASA's GISS Model-E uses the August equation, but with different coefficients

```
log10Pv = A - B/(C + T) // Antoine equation  
1888  
log10Pv = A - B/T // August equation ~1825
```

Ernst Ferdinand August (1795-1870) [coined the term psychrometer in 1818](#). In 1825, he invented a wet bulb / dry bulb psychrometer and [produced a formula to compute relative humidity](#) from those temperatures. From the references, it is not clear when he produced the saturation water vapor formula described here, but it had to be before his death (1870). The date in the table above (1825) should be considered an approximate guess based on the data available to me. The coefficients are fairly difficult to locate. The best I could do was [Physics Related to Anesthesia](#), By John D. Current, M.D. - the coefficients (on page 189) are for natural log, not base ten. The same coefficients are in [wikipedia](#), but without a reference. This equation has the same form as [the Clausius-Clapeyron relation](#) (~1850) assuming constant enthalpy (which implies constant specific heat) over the temperature range.

The Antoine equation (Antoine, C. - 1888) is based on the Clausius-Clapeyron relation (~1850). The coefficients are available [for various substances](#) (not just water) and they normally produce results as mmHg (torr). The program implements the algorithm with a single set of coefficients calibrated for the range of 0 to 100°C. To improve accuracy, [NIST](#) provides new coefficients about every 30°C.

This is the source code used in **mcSaturationVaporPressure.pas** - they both convert from mmHg to mbar.

Unfortunately, the best coefficients I could find are from wikipedia.

GISS Model-E

GISS Model-E - the NASA climate model used for IPCC AR4 and AR5 - contains the following code to compute the saturation mixing ratio. (Reference **QSAT** function in Model-E's **UTILDBL.f**)

```
GISS Model-E formula
// This is what is in the code - reformatted for readability
QSAT = 6.108 * MRAT * EXP(Latent_heat * (7.93252E-6 - 2.166847E-
3/temp_K))/Pressure
Latent_heat = 2.834E6 ice
            = 2.5 E6 water
MRAT = (mwat/mair) // ratio of molecular weight of water over molecular weight
of air
// used to convert mixing ratio to relative heat content
QSAT = Partial Pressure * MRAT / Total Pressure
// Partial pressure (mbar) at saturation - QSAT with MRAT and Pressure
removed
Psat = 6.108 * EXP(Latent_heat * (7.93252E-6 - 2.166847E-3/T_K))
```

After a little algebra, this is the code that computes the saturation partial pressure in mbar.

```
Psat := 6.108 * exp(2.5E6 * (7.93252E-6 - 2.166847E-3/T_K)) // Latent_heat
water = 2.5E6
Psat := 6.108 * exp(2.834E6*(7.93252E-6 - 2.166847E-3/T_K)) // Latent_heat ice
= 2.834E6
```

The following compares the "over water" Model-E equation with the August equation. (Code from the **mcSaturationVaporPressure** unit.)

```
Psat := 6.108 * exp(2.5E6 * (7.93252E-6 - 2.166847E-3/T_K)); // GISS Model-E
AR4 & AR5
Psat := 1013.25/760 * exp(20.386 - 5132 / T_K); // August equation
```

The coefficients are significantly different because one produces results in mbar and the other in torr (mmHg). Using a little algebra and rearranging the terms

```

2.5E6 * 7.93252 E-6 = 19.8313
2.5E6 * 2.166847E-3 = 5417.1175
ln (6.108) = 1.8096
ln (1013.25/760) = 0.2876
ln (6.108) + 19.8313 = 21.6409
ln (1013.25/760) + 20.386 = 20.6736
Psat := exp(21.6409 - 5417.1175 / T_K); // GISS Model-E AR4 &
AR5
Psat := exp(20.6736 - 5132 / T_K); // August equation

```

Obviously, the 2 equations have the same form, though the coefficients are still a bit different.

The Model-E saturation partial pressure at the triple point (6.108 mbar) is similar to that used by MagnusTetens (1967 - 6.1078 mbar) and Preining (2002 - 6.107 mbar). Even though [the current \(1976\) standard](#) is 6.11657 mbar at the triple point, [a 2005 JPL paper](#) claims

the saturation partial pressure of water vapor (over water) is 6.108 mbar at 0°C

Whether those values are close, or not, depends on the application .. but I expect better from both JPL and NASA. Either use the "correct" value, or explicitly say why you don't. At any rate, the following shows that the difference may, or may not, matter.

```

IR Absorption (computed with another program)
At 1013.25 mbar, 60.00 %RH, 1 km thick layer, 288.15 K (15°C) - surface
MurphyKoop 10,101 ppm 208.17 W/m2
GISS_AR4 10,148 ppm 208.31 W/m2
At 1013.25 mbar, 60.00 %RH, 1 km thick layer, 310.93 K (37.78°C 100°F) -
surface, hot
MurphyKoop 38,808 ppm 336.78 W/m2
GISS_AR4 40,229 ppm 338.17 W/m2
At 500 mbar 100 %RH, 1 km thick layer, 251.8 K (-21.20°C) - mid-troposphere
MurphyKoop 2,233 ppm 73.400 W/m2
GISS_AR4 2,272 ppm 73.669 W/m2
At 226.06 mbar 100 %RH, 1 km thick layer, 216.5 K (-56.65°C) - tropopause - 11
km
MurphyKoop 126 ppm 12.343 W/m2 - using over water formulas
GISS_AR4 151 ppm 13.260 W/m2
Using the over ice formulas, both algorithms give identical results at -
56.65°C
GISS_AR4 75 ppm 9.9554 W/m2

```

[Alduchov, et al](#) explain that thermometer values are rounded to the nearest tenth of a degree and, therefore, formulas with enough overlap due to the temperature round off error are basically the same. At 15C, that is about 108 ppm or 0.32%RH.

In the source code, NASA did not include the original date that they started using the algorithm, or a reference. (In other parts of the code, they give both.) Based on other information, such as dates in other source modules, their algorithm, with those constants, was probably first used from somewhere between 1960 and 1990 - but that is still a guess. Both algorithms were used in both AR4 and AR5 - 2007 and 2014.

By the way, this is the second algorithm in [NASA's Model-E](#) that I have had problems with - the version of the model used for IPCC AR4 used the wrong algorithm to compute the position of the Earth with respect to the Sun! (The correct algorithm was used for AR5.)

Seinfeld and Pandis

This equation is from [Atmospheric Chemistry and Physics](#) by Seinfeld and Pandis (2006). It is obviously calibrated for a boiling point of 100°C (373.15 K) at 1 atm (1013.25 mbar).

```
vpf_Seinfeld_Pandis: begin // over water only
  // Source : Atmospheric Chemistry and Physics by Seinfeld and Pandis
  a := 1 - (373.15/T_K);
  Psat := 1013.25 * exp((13.3185 - (1.97 + (0.6445 +
(0.1299*a))*a)*a)*a);
end;
```

I have additional textbooks, each with its own "best algorithm". I chose this one because I used it in an earlier program before I knew there were so many algorithms and because it clearly intersects the pre-1990 boiling point.

The other 25 equations

I have provided detailed discussions of 5 equations above, including the actual code used in **mcSaturationVaporPressure.pas**. The details of the other 25 are provided in the excellent paper by [Holger Vömel](#) (CIRES, University of Colorado, Boulder).

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